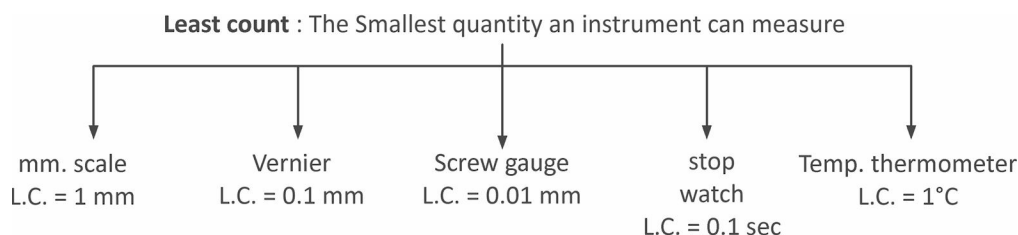


Experiments & Errors

Least Count :

No measurement is perfect. Every instrument can measure upto a certain accuracy ; called least count.



Permissible Error :

Error in measurement due to the limitation (least count) of the instrument, is called permissible error. From mm scale, we can measure upto 1 mm accuracy (least count). From this we will get measurement like $\ell = 34 \text{ mm}$.

Max uncertainty can be 1 mm.

Max permissible error ($\Delta\ell$) = 1mm

But if from any other instrument, we get $\ell = 34.5 \text{ mm}$, then max permissible error ($\Delta\ell$) = 0.1 mm and if from a more accurate instrument we get $\ell = 34.527 \text{ mm}$, then max permissible error ($\Delta\ell$) = 0.001 mm = place value of last number.

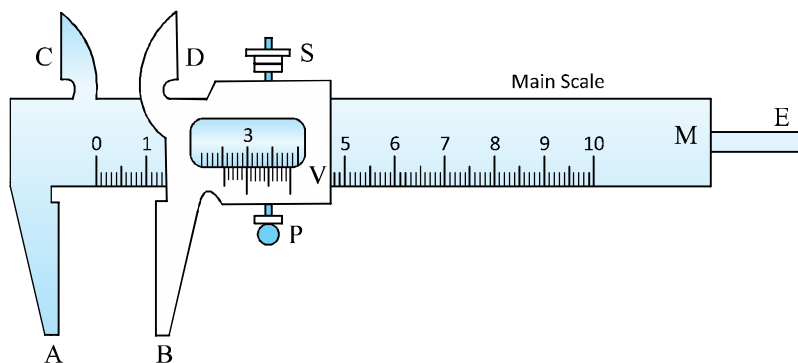
Max permissible error in a measured quantity is equal to place value of the last number

VERNIER CALLIPERS

Experiment - 1

It has three parts.

(i) **Main Scale** : It consists of a steel metallic strip M, graduated in cm and mm at one edge.



It carries two fixed jaws A and C as shown in figure.

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Illustration - 1 The smallest division on main scale of a vernier callipers is 1 mm and 10 vernier divisions coincide with 9 main scale divisions. While measuring the length of a line, the zero mark of the vernier scale lies between 10.2 cm and 10.3 cm and the third division of vernier scale coincide with a main scale division.

- (a) Determine the least count of the callipers (b) Find the length of the line

SOLUTION :

(a) Least count (L.C.) = 1MSD – 1VSD = 1 mm – 0.9 mm = 0.1 mm = 0.01 cm

(b) $L = N + n (\text{L.C.}) = (10.2 + 3 \times 0.01) \text{ cm} = 10.23 \text{ cm}$

Illustration - 2 N -divisions on the main scale of a vernier callipers coincide with $N + 1$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument.

SOLUTION :

$(N + 1)$ divisions on the vernier scale = N divisions on main scale

$\therefore$ 1 division on vernier scale = $\frac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

$\therefore$ 1 division on vernier scale = $\left(\frac{N}{N+1}\right) a$ units = a' (say)

Least count = 1 main scale division – 1 vernier scale division = $a - a' = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$ units.

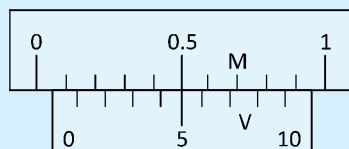
Zero Error :

If the zero of the vernier scale does not coincide with the zero of main scale when jaw B touches A and the straight edge of D touches the straight edge of C, then the instrument has an error called zero error. Zero error is always algebraically subtracted from measured length.

Zero error $\longrightarrow$ algebraically subtracted

If zero of vernier scale lies to the right of the main scale the zero error is positive and if it lies to the left of the main scale the zero error is negative (when jaws A and B are in contact).

Illustration - 3 In the diagram shown in figure, find the magnitude and nature of zero error.



SOLUTION :

Here zero of vernier scale lies to the right of zero of main scale, hence, it has positive zero error.

Further, $N = 0, x = 5, \text{L.C. or V.C.} = 0.01 \text{ cm}$

Hence, Zero error $= N + x \times \text{V.C.} = 0 + 5 \times 0.01 = 0.05 \text{ cm}$

$\therefore$ Actual length will be 0.05 cm less than the measured length.

Illustration - 4

Least count of a vernier callipers is 0.01 cm. When the two jaws of the instrument touch each other, the 5th division of the vernier scale coincide with a main scale division and the zero of the vernier scale lies to the left of the zero of the main scale. Furthermore while measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.4 cm and 2.5 cm and the 6th vernier division coincides with a main scale division. Calculate the diameter of the sphere.

SOLUTION :

The instrument has a negative zero error,

$$e = -1\text{cm} + 5 \times 0.01 \text{ cm} \quad \text{or} \quad e = -0.05 \text{ cm}$$

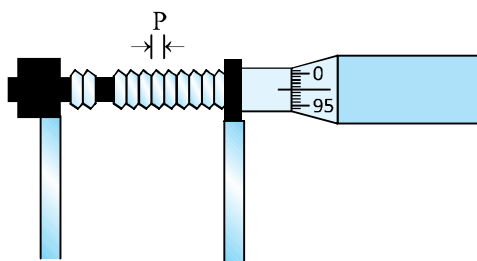
Measured reading $= (2.4 + 6 \times 0.01) = 2.46 \text{ cm}$

True reading $= \text{Measured reading} - e = 2.46 - (-0.05) = 2.51 \text{ cm}$

Therefore, diameter of the sphere is 2.51 cm.

SCREW GAUGE**Experiment - 2**

The least count of vernier callipers ordinarily available in the laboratory is 0.01 cm. When lengths are to be measured with greater accuracy, say upto 0.001 cm, screw gauge is used which is based on the principle of micrometer screw discussed below.



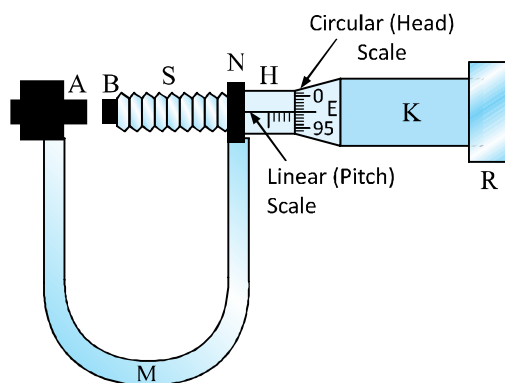
If an accurately cut single threaded screw is rotated in a closely fitted nut, then in addition to the circular motion of the screw there is a linear motion of the screw head in the forward or backward direction, along the axis of the screw. The linear distance moved by the screw, when it is given one complete rotation is called the pitch (p) of the screw. This is equal to the distance between two consecutive threads as measured along the axis of the screw. In most of the cases it is either 1 mm or 0.5 mm. A circular cap is fixed on one end of the screw and the circumference of the cap is normally divided into 100 or 50 equal parts. If it is divided into 100 (or 50) equal parts, then the screw moves forward or backward by $\frac{1}{100}$ (or $\frac{1}{50}$) of the pitch, if the circular scale (we will discuss later about circular scale) is rotated through one circular scale division. It is the minimum distance which can be accurately measured and so called the least count (L.C.) of the screw.

$$\text{Thus, Least count} = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

If pitch is 1 mm and there are 100 divisions on circular scale then, $\text{L.C.} = \frac{1\text{ mm}}{100} = 0.01\text{ mm} = 0.001\text{ cm} = 10\text{ }\mu\text{m}$

Since, L.C. is of the order of $10\text{ }\mu\text{C}$, the screw is called micrometer screw.

Screw gauge works on the principle of micrometer screw. It consists of a U-shaped metal frame M. At one end of it is fixed a small metal piece A. It is called stud and it has a plane face. The other end N of M carries a cylindrical hub H. It is graduated in millimeters and half millimetre depending upon the pitch of the screw. This scale is called linear scale or pitch scale.



A nut is threaded through the hub and the frame M. The front face B of the screw, facing the plane face A capable of rotating over the hub when screw is rotated. As the cap is rotated the screw either moves in or out. The surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or head scale. In an accurately adjusted instrument when the faces A and B are just touching each other. Zero of circular scale should coincide with zero of linear scale.

To Measure the diameter of a given Wire using a Screw Gauge :

If with the wire between plane faces A and B, the edge of the cap lies ahead of Nth division of linear scale, and nth division of circular scale lies over reference line.

Then, $\text{Total reading} = N + n \times \text{L.C.}$

Illustration - 5 The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are six divisions on the linear scale and forty divisions on circular scale coincides with the reference line. Find the diameter of the sphere.

SOLUTION :

$$\text{L.C.} = \frac{1}{100} = 0.01\text{ mm}$$

$$\text{Linear scale reading} = 6 (\text{pitch}) = 6\text{ mm}$$

$$\text{Circular scale reading} = n(\text{L.C.}) = 40 \times 0.01 = 0.4\text{ mm}$$

$$\therefore \text{Total reading} = (6 + 0.4) = 6.4\text{ mm}$$

Illustration - 6 A screw gauge of pitch 1 mm has a circular scale divided into 100 divisions. The diameter of a wire is to be measured by above said screw gauge. The main scale reading is 1 mm and 47th circular division coincides with main scale. Find the curved surface area of wire in true significant figures. (Given the length of wire is equal to 5.6 cm and there is no zero-error in the screw gauge).

SOLUTION :

$$\text{Least count} = \frac{1\text{mm}}{100} = 0.01\text{mm}$$

$$\begin{aligned}\text{Diameter} &= \text{M.S.R.} + \text{Number of division coinciding with main scale} \times \text{Least count.} \\ &= 1\text{mm} + 47 \times 0.01\text{ mm} = 1.47\text{ mm} = 0.147\text{ cm}\end{aligned}$$

$$\text{Curved surface area} = \pi d \ell = \frac{22}{7} \times 0.147 \times 5.6 = 2.6\text{ cm}^2$$

Zero Error :

If zero mark of circular scale does not coincide with the zero of the pitch scale when the faces A and B are just touching each other, the instrument is said to possess zero error. If the zero of the pitch scale advances beyond the cap, the zero error is positive. If it is inside, the zero error is negative.

Illustration - 7 The pitch of a screw gauge is 1 mm and there are 100 divisions on circular scale. When faces A and B are just touching each without putting anything between the studs 32nd division of the circular scale coincides with the reference line. When a glass plate is placed between the studs, the linear scale reads 4 divisions and the circular scale reads 16 divisions. Find the thickness of the glass plate. Zero of linear scale is not hidden from circular scale when A and B touch each other.

SOLUTION :

$$\begin{aligned}\text{Least count L.C.} &= \frac{\text{pitch}}{\text{Number of divisions on circular scale}} = \frac{1}{100}\text{ mm} \\ &= 0.01\text{ mm}\end{aligned}$$

As zero is not hidden from circular scale when A and B touches each other. Hence, the screw gauge has positive error.

$$e = + n (\text{L.C}) = 32 \times 0.01 = 0.32\text{ mm}$$

$$\text{Linear scale reading} = 4 \times (1\text{ mm}) = 4\text{ mm}$$

$$\text{Circular scale reading} = 16 \times (0.01\text{ mm}) = 0.16\text{ mm}$$

$$\therefore \text{Measured reading} = (4 + 0.16)\text{ mm} = 4.16\text{ mm}$$

$$\therefore \text{Absolute reading} = \text{Measured reading} - e = (4.16 - 0.32)\text{ mm} = 3.84\text{ mm}$$

Therefore, thickness of the glass plate is 3.84 mm

The pitch of a screw gauge is 1 mm and there are 100 divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 6 divisions below the reference line. When a wire is placed between the jaws, 2 linear scale divisions are clearly visible while 62 divisions on circular scale coincide w

Illustration - 9 In an experiment to determine the time period of a simple pendulum, a student uses a watch with least count 1 second and measures the time taken for 10 oscillations to be 37 seconds. Then, the time period of the pendulum that he should report is :

- (A) $3.7 \pm 1.0 \text{ sec}$ (B) $3.7 \pm 0.1 \text{ sec}$ (C) $3.70 \pm 1.00 \text{ sec}$ (D) $3.70 \pm 0.10 \text{ sec}$

SOLUTION : (B)

Total time for 10 oscillations, $t = 37 \pm 1 \text{ sec}$

Therefore, the time period, $T = \frac{1}{10}t = 3.7 \pm 0.1 \text{ sec}$

Illustration - 10 In an experiment to determine the value of g using a simple pendulum, the measured length of the pendulum string is 40.0 cm, and the measured time taken for 10 oscillations is 14.0 sec. Find the percentage error in the determined value of g . (Take $\pi = \frac{25}{8}$)

SOLUTION :

Length of the string,

$$L = 40.0 \text{ cm} \quad \text{and} \quad \Delta L = 0.1 \text{ cm}$$

Time period of the pendulum,

$$T = \frac{14}{10} = 1.4 \text{ s} \quad \text{and} \quad \Delta T = \frac{0.1}{10} = 0.01 \text{ s}$$

We know that $g = 4\pi^2 \left(\frac{L}{T^2} \right)$

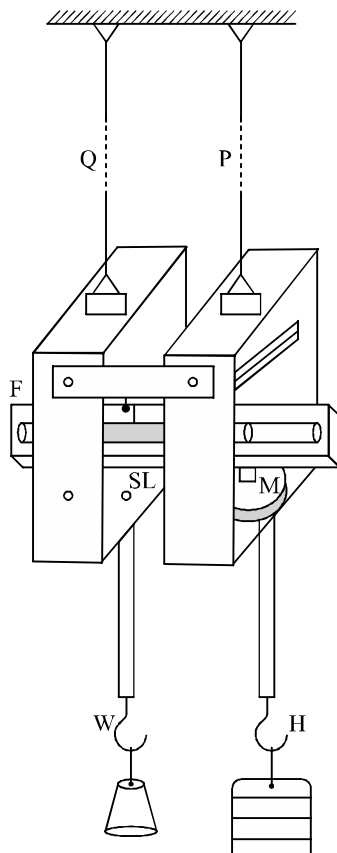
Therefore, $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \left(\frac{\Delta T}{T} \right) = \frac{0.1}{40} + 2 \left(\frac{0.01}{1.4} \right) = 1.68 \%$

DETERMINATION OF YOUNG'S MODULUS BY SEARLE'S METHOD

Experiment - 4

The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the side of the experimental wire. On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level.

In some situations, the change in length is obtained by vernier arrangement instead of the screw gauge.

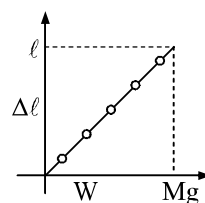


From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin.

The slope of the line gives $\tan \theta = \frac{l}{W} = \frac{l}{Mg}$

Now, stress = $\frac{Mg}{\pi r^2}$ and strain = $\frac{l}{L}$

$$Y = \text{Stress/strain} = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi r^2 \tan \theta}$$



With known values of initial length L , radius r of the experimental wire and $\tan \theta$, Young's modulus Y can be calculated. Error arising due to measurement of L , l and r is given by :

$$\left| \frac{\Delta y}{y} \right| = \left| \frac{\Delta L}{L} \right| + 2 \left| \frac{\Delta r}{r} \right| + \left| \frac{\Delta l}{l} \right|$$

Illustration - 11

In Searle's apparatus, diameter of the wire was measured as 0.05 cm by screw gauge of least count 0.001 cm. The length of wire was measured 110 cm by meter scale of least count 0.1 cm. An external load of 50 N was applied. The extension in length of wire was measured 0.125 cm by micrometer of least count 0.001 cm. Find the maximum possible error in measurement of young's modulus.

SOLUTION :

$$Y = \frac{4F / \pi D^2}{\Delta L / L} = \frac{4FL}{\pi D^2 (\Delta L)}$$

Maximum possible relative error

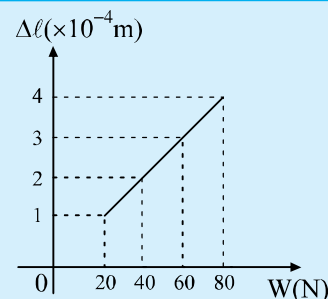
$$\frac{\Delta Y}{Y} = \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta(\Delta L)}{\Delta L} = \left(\frac{0.1}{110} + \frac{2 \times 0.001}{0.050} + \frac{0.001}{0.125} \right)$$

Percentage error

$$100 \times \frac{\Delta Y}{Y} = \frac{1}{11} + 4 + \frac{4}{5} = 0.09 + 4 + 0.8 = 4.89\%$$

Illustration - 12 The adjacent graph shows the extension ($\Delta \ell$) of a wire of length 1m suspended from the top of a roof at one end, with an extra load Δw connected to the other end. If the cross sectional area of the wire is 10^{-6}m^2 , calculate the Young's modulus of the material of the wire.

- (A) $2 \times 10^{11} \text{ N/m}^2$ (B) $2 \times 10^{-11} \text{ N/m}^2$
 (C) $3 \times 10^{13} \text{ N/m}^3$ (D) $2 \times 10^{16} \text{ N/m}^2$

**SOLUTION : (A)**

$$\Delta l = \left(\frac{l_0}{AY} \right) \Delta w;$$

$$\text{Slope} = \frac{l_0}{AY} = \frac{1 \times 10^{-4}}{20} = \frac{1}{(10^{-6})Y}$$

$$Y = 20 \times 10^{10} = 2 \times 10^{11} \text{ N/m}^2$$

DETERMINATION OF SPECIFIC HEAT OF A LIQUID USING CALORIMETER**Experiment - 5**

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) through a heating coil immersed within the liquid for a known length of time (t).

The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to be θ_2 .

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1) S_l (\theta_2 - \theta_0) + m_1 S_c (\theta_2 - \theta_0) = i \cdot V \cdot t \quad \text{or} \quad (m_2 - m_1) S_l + m_1 S_c = i \cdot V \cdot t / (\theta_2 - \theta_0) \quad \dots (i)$$

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown : one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate $m_1 S_c$.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

Illustration - 13 In this experiment, voltage across the heater is 100.0 V and current is 10.0 A , and heater was switched on for $t = 700.0\text{ sec}$. Initially all elements were at room temperature, $\theta_0 = 10.0^\circ\text{C}$ and final temperature was measured as $\theta_f = 80.0^\circ\text{C}$. Mass of empty calorimeter was 1.0 kg and the combined mass of calorimeter + liquid is 3.0 kg . The specific heat capacity of the calorimeter is $S_c = 3.0 \times 10^3\text{ J/kg}^\circ\text{C}$. Find the specific heat capacity of the unknown liquid in proper significant figures.

(A) $3.5 \times 10^3\text{ J/kg}^\circ\text{C}$

(B) $3.50 \times 10^3\text{ J/kg}^\circ\text{C}$

(C) $4.0 \times 10^3\text{ J/kg}^\circ\text{C}$

(D) $3.500 \times 10^3\text{ J/kg}^\circ\text{C}$

SOLUTION : (A)

$$S_\ell = \frac{\frac{(100.0)(10.0)(700.0)}{80.0 - 10.0} - (1.0)(3.0 \times 10^3)}{3.0 - 1.0}$$

$$= 3.5 \times 10^3\text{ J/Kg}^\circ\text{C} \text{ (According to addition and multiplication rule of S.F)}$$

Illustration - 14 If mass and specific heat capacity of calorimeter is negligible, what would be maximum permissible error in S_ℓ . Use the data mentioned below :

$$m_1 \rightarrow 0, S_c \rightarrow 0, m_2 = 1.00\text{ kg}, V = 10.0\text{ V}, I = 10.0\text{ A}, t = 1.00 \times 10^2\text{ sec}, \theta_0 = 15^\circ\text{C}, \text{ Corrected } \theta_f = 65^\circ\text{C}$$

(A) 4%

(B) 5%

(C) 8 %

(D) 12%

SOLUTION : (C)

$$\text{If } m_1 \rightarrow 0, S_c \rightarrow 0$$

$$S_\ell = \frac{VIt}{m_2(\theta_f - \theta_0)}$$

$$\frac{dS_\ell}{S_\ell} = \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta t}{t} + \frac{\Delta m_2}{m_2} + \frac{\Delta \theta_f + \Delta \theta_0}{\theta_f - \theta_0} = \frac{0.1}{10.0} + \frac{0.1}{10.0} + \frac{0.01 \times 10^2}{1.00 \times 10^2} + \frac{0.01}{1.00} + \frac{1+1}{50} = 8\%$$

FOCAL LENGTH OF A CONCAVE MIRROR & A CONVEX LENS USING THE U-V METHOD

Experiment - 6

In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder.

The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

A plot of $|u|$ versus $|v|$ gives a rectangular hyperbola. A plot of $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line.

The error in this experiment is mostly due to improper position of the object on the holder.

The error due to measurement's given by : $\frac{|\Delta f|}{f^2} = \frac{|\Delta u|}{u^2} + \frac{|\Delta v|}{v^2}$

The errors Δu , Δv correspond to the error in the measurement of u and v .

Illustration - 15 In $u-v$ method to find focal length of a concave mirror, if object distance was found to be 10.0 cm and image distance was also found to be 10.0 cm then find the focal length with error limits.

SOLUTION :

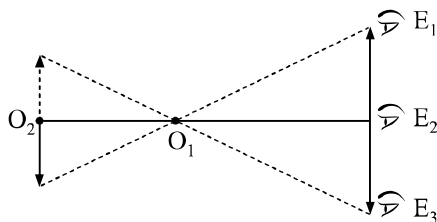
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{(-10)} + \frac{1}{(-10)} = \frac{1}{f} \Rightarrow |f| = 5\text{cm}$$

$$(df)_{\max} = \left(\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right) \times f^2$$

$$(df)_{\max} = \left(\frac{0.1}{10^2} + \frac{0.1}{10^2} \right) \times 5^2 = 0.05\text{ cm} \quad \text{so,} \quad f = (5 \pm 0.05)\text{cm}$$

Parallax :

When two objects O_1 and O_2 are placed in such a way that both of them lie in the same line of sight as shown in figure, then the object nearer to the eye covers the object farther from it. Their images on the retina are superimposed and therefore, it is impossible to decide which is the nearer object. To identify this fact, the observer displaces his eye to the position E_1 or E_2 until he is able to see two distinct objects.



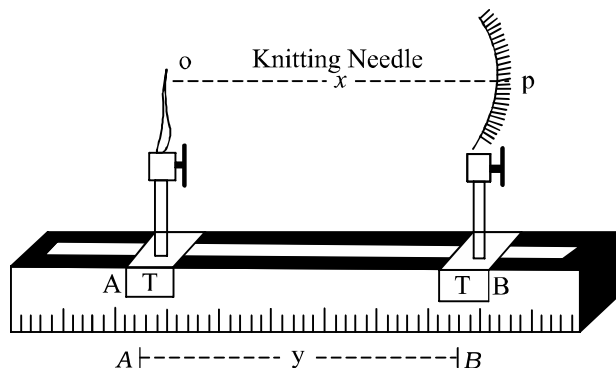
The more distant object O_2 apparently moves in the direction opposite to the displacement of the observer's eye with respect to the nearer object O_1 . This relative shift in the position of two objects due to the shift in the position of the observer's eye is called parallax.

Index Error :

In $u-v$ method, we require the distance between object or image from the pole (vertex) of the mirror (actual distance).

But practically we measure the distance between the indices A and B . Observed distance, which need not exactly coincide with object and pole, there can be a slight mismatch called index error, which will be constant for every observation.

Index error = Observed distance – Actual distance (Just like zero error in screw gauge, it is the excess reading). To determine index error, mirror and object needle are placed at arbitrary position. For measuring actual distance, a knitting needle is just fitted between the pole of mirror and object needle “O”. The length of knitting needle will give the actual object distance while the separation between indices A and B at that instant is the observed distance.



Determination of index correction.

So index error is

$$e = \text{Observed distance} - \text{Actual distance}$$

$$= \text{Separation between indices } A \text{ and } B - \text{Length of knitting needle}$$

Once we get e , in every observation, we will use :

$$\text{Actual distance} = \text{Observed distance (separation between the indices)} - \text{Excess reading } (e)$$

There is another term, Index correction which is negative of index error.

$$\text{Index correction} = -\text{index error.}$$

Illustration - 16 To find index error for u , when a knitting needle of length 20.0 cm is adjusted between pole and object needle, the separation between the indices of object needle and mirror was observed to be 20.2 cm.

- (a) Index error for u is:
 (A) -0.2 cm (B) 0.2 cm (C) -0.1 cm (D) 0.1 cm
- (b) To find index error for v , when the same knitting needle is adjusted between the pole and the image needle, the separation between the indices of image needle and mirror was found to be 19.9 cm. Index error for v is:
 (A) 0.1 cm (B) -0.1 cm (C) 0.2 cm (D) -0.2 cm
- (c) In same observation, the observed object distance (Separation between indices of object needle and mirror) is 30.2 cm, and the observed image distance is 19.9 cm. Using index correction from previous two questions, estimate the focal length of the concave mirror.

SOLUTION :

(a)-(B) Index error (Excess reading) = Observed reading – Actual reading

$$= 20.2 - 20.0 = 0.2 \text{ cm}$$

(b)-(B) $e = 19.9 \text{ cm} - 20.0 \text{ cm} = -0.1 \text{ cm}$

$$\begin{aligned}
 \text{(c)} \quad u &= 30.2 - 0.2 \text{ (excess reading)} \\
 &= 30.0 \text{ cm} \\
 v &= 19.9 - (-0.1) \text{ (excess reading)} \\
 &= 20.0 \text{ cm.}
 \end{aligned}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow f = 12.0 \text{ cm.}$$

DETERMINATION OF SPEED OF SOUND USING RESONANCE COLUMN

Experiment - 7

Figure shows the experiment to find velocity of sound in air using Resonance tube method.

Principle : Resonance tube is a closed organ pipe. So the series of frequencies at which it can vibrate is :

$$\frac{V}{4l_{eq}}, \frac{3V}{4l_{eq}}, \frac{5V}{4l_{eq}}, \dots$$

or Generally $f_n = (2n-1) \frac{V}{4l_{eq}}$

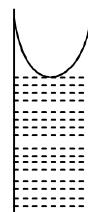
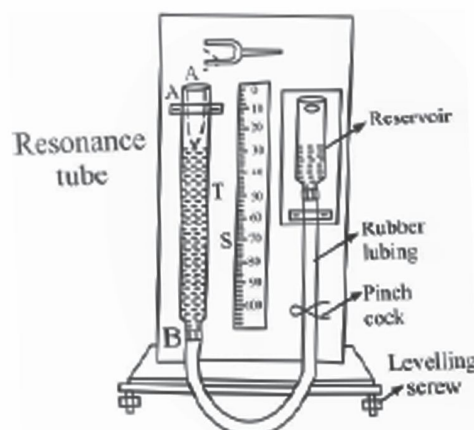
If the tube resonates with a tuning fork of frequency f_0 ;

$$(2n-1) \frac{V}{4l_{eq}} = f_0$$

$$\Rightarrow l_{eq} = (2n-1) \frac{V}{4f_0}$$

For the first Resonance $l_{eq} = \frac{V}{4f_0}$ = (corresponding to 1st mode)

For the second Resonance $l_{eq} = \frac{3V}{4f_0}$ = (corresponding to 2nd mode)



Working : Resonance tube is a 100 cm tube. Initially it is filled with water. To increase the length of air column in the tube, water level is lowered. The air column is forced with a tuning of frequency f_0 . Suppose at length l_1 , we get a first resonance (loud voice) then,

$$l_{eq1} = \frac{V}{4f_0}$$

$$\Rightarrow l_1 + \epsilon = \frac{V}{4f_0} \quad \dots(i)$$

If we further lower the water level, the noise becomes moderate. But at l_2 , we again get a loud noise (second resonance) then

$$l_{eq2} = \frac{3V}{4f_0}$$

$$\Rightarrow l_2 + \varepsilon = \frac{3V}{4f_0} \quad \dots(ii)$$

From (i) and (ii) $V = 2f_0(l_2 - l_1)$

Errors : The error introduced are due to end effects (end correction) and also due to excessive humidity.

error due to measurement is given by $\frac{|\Delta V|}{V} = \frac{\Delta(l_2 - l_1)}{l_2 - l_1} = \frac{|\Delta l_2| + |\Delta l_1|}{l_2 - l_1}$

Illustration - 17 Observation table :

Room temperature in beginning = 26°C

Room temperature at end = 28°C

		Position of water level (cm)			
Freq. of tuning fork in (Hz) (f_0)	Resonance	Water level is falling	Water level is rising	Mean resonant length	Speed of sound $V = 2f_0(l_2 - l_1)$
340 Hz	1st Resonance	23.9	24.1	$l_1 = \dots\dots\dots$	$V = \dots\dots\dots$
	2nd Resonance	73.9	74.1	$l_2 = \dots\dots\dots$	

- (a) Speed of sound calculated is roughly :
- (A) 340 m/sec (B) 380 m/sec (C) 430 m/sec (D) None of these
- (b) In the previous question, speed of sound at 0° is roughly.
- (A) 324 m/sec (B) 380 m/sec (C) 430 m/sec (D) None of these
- (c) What should be minimum length of tube, so that third resonance can also be heard ?
- (A) $l_3 = 421$ (B) $l_3 = 214$ (C) $l_3 = 124$ (D) None of these
- (d) Estimate the diameter of the tube using empirical formula ($\varepsilon \approx 0.3d$).
- (A) 2.5 cm (B) 3.3 cm (C) 5.2 cm (D) None of these

SOLUTION :

(a)-(A) $l_1 = 24.0 \text{ cm}$

$l_2 = 74.0 \text{ cm}$

$V = 2f_0(l_2 - l_1) = 2(340)(0.740 - 0.240) = (2)(340)(0.500) = 340 \text{ m/sec.}$

(b)-(A) $v \propto \sqrt{T} \quad \frac{V_{27^\circ}}{V_{0^\circ}} = \sqrt{\frac{300}{273}}$

$V_{0^\circ} = V_{27^\circ} \sqrt{\frac{273}{300}} = 340 \sqrt{\frac{273}{300}} = 324 \text{ m/sec.}$

$$(c)-(C) \quad l_1 + \varepsilon = \frac{V}{4f_0} \quad l_2 + \varepsilon = \frac{3V}{4f_0}$$

Solve both equations and get : $\varepsilon = 1 \text{ cm}$

$$\text{For third resonance, } l_3 + \varepsilon = \frac{5V}{4f_0}$$

$$\Rightarrow l_3 = 124 \text{ cm}$$

$$(d)-(B) \quad \varepsilon = 1 \text{ cm} = 0.3 \text{ d} ; d = \frac{1 \text{ cm}}{0.3} = 3.3 \text{ cm}$$

Illustration - 18 If a tuning fork of $(340 \text{ Hz} \pm 1\%)$ is used in the resonance tube method, and the first and second resonance lengths are 24.0 cm and 74.0 cm respectively. Find maximum permissible error in speed of sound.

SOLUTION :

$$l_1 = 24.0 \text{ cm} \rightarrow \Delta l_1 = 0.1 \text{ cm}$$

$$l_2 = 74.0 \text{ cm} \rightarrow \Delta l_2 = 0.1 \text{ cm}$$

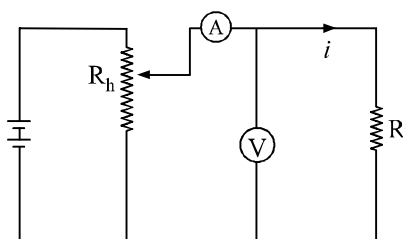
$$f_0 = (340 \text{ Hz} \pm 1\%) \Rightarrow \frac{\Delta f_0}{f_0} = 1\% = \frac{1}{100}$$

$$\left(\frac{dV}{V} \right)_{\max} = \frac{\Delta f_0}{f_0} + \frac{\Delta l_2 + \Delta l_1}{l_2 - l_1} = \frac{1}{100} + \frac{0.1 + 0.1}{50.0} = 0.014 \quad \text{Percentage error} = 1.4\%$$

VERIFICATION OF OHM'S LAW USING VOLTMETER & AMMETER

Experiment - 8

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh.



Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (R_h). A graph of V vs i is plotted and it is found to be linear (within errors). The magnitude of R is determined by either.

(a) taking the ratio V/i or (b) fitting the readings to a straight line : $V = iR$, and determining the slope R .

Errors : Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/connections of the resistance. The effect of Joule heating may be minimized by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimized as much as possible.

Errors Analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left| \frac{\Delta R}{R} \right| = \left| \frac{\Delta V}{V} \right| + \left| \frac{\Delta i}{i} \right|$ where ΔV and Δi are of the order of the least counts of the instruments used.

We can find the specific resistance of a material using ohm's law experiment

$$\rho = \frac{RA}{L} = \frac{\pi D^2}{4L} \frac{V}{I}$$

$$\ln \rho = \ln \frac{\pi}{4} + 2 \ln D - 2 \ln L + \ln V - \ln I$$

$$\frac{d\rho}{\rho} = 2 \frac{dD}{D} - \frac{dL}{L} + \frac{dV}{V} - \frac{dI}{I}$$

$$\frac{d\rho}{\rho} = \pm 2 \frac{\Delta D}{D} \mp \frac{\Delta L}{L} \pm \frac{\Delta V}{V} \mp \frac{\Delta I}{I}$$

$$\left(\frac{d\rho}{\rho} \right)_{\max} = \max \text{ of } \left(\pm 2 \frac{\Delta D}{D} \mp \frac{\Delta L}{L} \pm \frac{\Delta V}{V} \mp \frac{\Delta I}{I} \right)$$

$$\left(\frac{d\rho}{\rho} \right)_{\max} = + 2 \frac{\Delta D}{D} + \frac{\Delta L}{L} + \frac{\Delta V}{V} + \frac{\Delta I}{I} = \max. \text{ permissible error in } \rho$$

Illustration - 19

In the experiment of Ohm's law, when potential difference 10.0 V is applied, current measured is 1.00 A. If length of wire is found to be 10.0 cm, and diameter of wire is 2.50 mm, then the maximum permissible error in resistivity will be:

- (A) 1.8% (B) 10.2% (C) 3.8% (D) 5.75%

SOLUTION : (C)

$$\left(\frac{d\rho}{\rho} \right)_{\max} = 2 \left(\frac{0.01}{2.50} \right) + \left(\frac{0.1}{10.0} \right) + \left(\frac{0.1}{10.0} \right) + \left(\frac{0.01}{1.00} \right) = 3.8\%$$

Illustration - 20

In an Ohm's Law experiment, current measured is $i = 10.0$ Amp., potential difference measured is $V = 100.0$ V, length of wire is 31.4 cm, and diameter of wire is 2.00 mm (all in correct significant figures).

(a) The resistivity of wire (in correct significant figure) will be : (Use $\pi = 3.14$)

- (A) $1.00 \times 10^{-4} \Omega\text{-m}$ (B) $1.0 \times 10^{-4} \Omega\text{-m}$
 (C) $1 \times 10^{-4} \Omega\text{-m}$ (D) $1.000 \times 10^{-4} \Omega\text{-m}$

(b) In the previous question, maximum permissible error in resistivity and resistance measurement will be : (respectively)

- (A) 2.14%, 1.5% (B) 1.5%, 2.45% (C) 2.41%, 1.1% (D) None of these

SOLUTION :

$$(a)-(A) \quad \rho = \frac{\pi D^2}{4L} \frac{V}{I} = \frac{(3.14)(2.00 \times 10^{-3})^2}{4(0.314)} \left(\frac{100.0}{10.0} \right)$$

and answer should be in three significant figures.

$$\text{So } \rho = 1.00 \times 10^{-4} \Omega - m$$

$$(b)-(C) \quad \left(\frac{dR}{R} \right)_{\max} = \frac{\Delta i}{i} + \frac{\Delta v}{v} = \frac{0.1}{10.0} + \frac{0.1}{100.0} = 1.1\%$$

$$\left(\frac{d\rho}{\rho} \right)_{\max} = 2 \left(\frac{0.01}{2.00} \right) + \left(\frac{0.1}{31.4} \right) + \left(\frac{0.1}{10.0} \right) + \left(\frac{0.1}{100.0} \right) = 2.41\%$$

SPECIFIC RESISTANCE OF THE MATERIAL OF A WIRE USING METER BRIDGE

Experiment - 9

A known length (l) of a wire is connected in one of the gaps (P) of a meter bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G). The balance length (l) is found by closing key K and momentarily connecting the galvanometer until it gives zero deflection (null point).

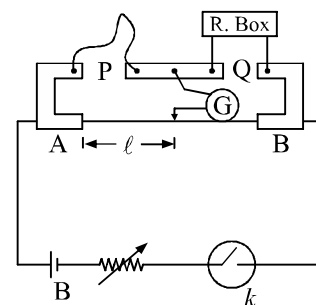
$$\text{Then, } \frac{P}{Q} = \frac{l}{100-l} \quad \dots(i)$$

Using the expression for the meter bridge at balance. Here P represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire,

$$P = \rho \frac{L}{\pi r^2} \Rightarrow \rho = \frac{\pi r^2}{L} P = \frac{\pi r^2}{L} \frac{Ql}{(100-l)} \quad \dots(ii)$$

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.



Errors : The major errors in this experiment are due to the heating effect, end corrections introduced due to shift of the zero of the scale at A and B, and errors due to non-uniformity of the meter bridge wire.

$$\text{The error in } \rho \text{ is } \frac{|\Delta \rho|}{\rho} = \frac{|2\Delta r|}{r} + \frac{|\Delta L|}{L} + \frac{|\Delta l|}{l} + \frac{\Delta l}{(100-l)} \quad (\text{If } Q \text{ is known precisely})$$

$$\square \quad \left(\frac{d\rho}{\rho} \right)_{\max} \text{ due to error in } l \text{ only is } \frac{\Delta l}{l} + \frac{\Delta l}{100-l} = \frac{\Delta l(100)}{l(100-l)}$$

$$\square \quad \left(\frac{d\rho}{\rho} \right)_{\max} \text{ will be least when } l(100-l) \text{ is maximum, i.e., } l = 50 \text{ cm}$$

Illustration - 21 In a meter bridge experiment, a wire (R) of diameter 1 mm, length 31.4 cm and unknown resistivity is connected in the left gap and a known resistance ($S = 300 \Omega$) is connected in the right gap. The balancing point is found at a distance 25.0 cm from the end A of the meter bridge wire AB.

- (a) The resistivity of the wire R is :
- (A) $2.5 \times 10^{-4} \Omega\text{-m}$ (B) $3.5 \times 10^{-4} \Omega\text{-m}$
 (C) $4.5 \times 10^{-4} \Omega\text{-m}$ (D) None of these
- (b) In the previous question, if R and S are interchanged, the balanced point is shifted by:
- (A) 30 cm (B) 40 cm (C) 50 cm (D) None of these

SOLUTION :

$$(a)-(A) \quad \frac{R}{300} = \frac{2.5}{75} \Rightarrow R = 100; \quad \rho = \frac{R\pi d^2}{4L} = 2.5 \times 10^{-4} \Omega\text{-m}$$

(b)-(C) If R and S are interchanged $\ell = 75, 100 - \ell = 25$
 Balanced point will be shifted by $75 - 25 = 50 \text{ cm}$

End Corrections :

In meter Bridge circuit, some extra length comes (is found under metallic strips) at end point A and B. So some additional length (α and β) should be included at ends for accurate result. Hence in place of ℓ , we use $\ell + \alpha$ and in place of $100 - \ell$, we use $100 - \ell + \beta$ (where α and β are called end correction). To estimate α and β , we use known resistance R_1 and R_2 at the place of R and S in meter Bridge. Suppose we get null point at ℓ_1 distance, then

$$\frac{R_1}{R_2} = \frac{\ell_1 + \alpha}{100 - \ell_1 + \beta} \quad \dots\dots(i)$$

Now we interchange the position of R_1 and R_2 , and get null point at ℓ_2 distance, then

$$\frac{R_2}{R_1} = \frac{\ell_2 + \alpha}{100 - \ell_2 + \beta} \quad \dots\dots(ii)$$

Solving equation (i) and (ii), we get :

$$\alpha = \frac{R_2 \ell_1 - R_1 \ell_2}{R_1 - R_2} \quad \text{and} \quad \beta = \frac{R_1 \ell_1 - R_2 \ell_2}{R_1 - R_2} - 100$$

These end corrections (α and β) are used to modify the observations

Illustration - 22 (a) If we use 100Ω and 200Ω resistance in place of R and S , we get null deflection at $\ell_1 = 33.0 \text{ cm}$. If we interchange the resistance, the null deflection was found to be at $\ell_2 = 67.0 \text{ cm}$. The end correction α and β should be :

- (A) $\alpha = 1 \text{ cm}, \beta = 1 \text{ cm}$ (B) $\alpha = 2 \text{ cm}, \beta = 1 \text{ cm}$
 (C) $\alpha = 1 \text{ cm}, \beta = 2 \text{ cm}$ (D) None of these

- (b) Now we start taking observations. At the position of R , unknown resistance is used and at position of S , 300Ω resistance is used. If the balanced length was found to be $\ell = 26\text{cm}$, estimate the unknown resistance.
- (A) 108Ω (B) 105.4Ω (C) 100Ω (D) 110Ω
- (c) If the unknown Resistance calculated without using the end correction, is R_1 and with using the end corrections is R_2 then:
- (A) $R_1 > R_2$ when balanced point is in first half (B) $R_1 < R_2$ when balanced point is in first half
 (C) $R_1 > R_2$ when balanced point is in second half (D) $R_1 > R_2$ always

SOLUTION :

$$(a)-(A) \quad \alpha = \frac{R_2 \ell_1 - R_1 \ell_2}{R_1 - R_2} = \frac{(200)(33) - (100)(67)}{100 - 200} = 1\text{cm}$$

$$\beta = \frac{R_1 \ell_1 - R_2 \ell_2}{R_1 - R_2} - 100 = \frac{(100) - (200)(67)}{100 - 200} - 100 = 1\text{cm}$$

$$(b)-(A) \quad \frac{\ell_{eq}}{(100 - \ell)_{eq}} = \frac{R}{300}$$

$$\Rightarrow \frac{R}{(300)} = \frac{26 + 1}{(100 - 26) + 1} = \frac{27}{75} \Rightarrow R = \frac{300 \times 27}{75} = 108\Omega$$

$$(c)-(C) \quad R_1 = S \left(\frac{\ell}{100 - \ell} \right), \quad R_2 = S \left(\frac{\ell + \alpha}{100 - \ell + \beta} \right)$$

Let us find the quantity $R_1 - R_2$

Using the values $\alpha = \beta = 1$,

$$R_1 - R_2 = S \left(\frac{l}{100 - l} - \frac{l + 1}{101 - l} \right) = S \left(\frac{2l - 100}{(100 - l)(101 - l)} \right)$$

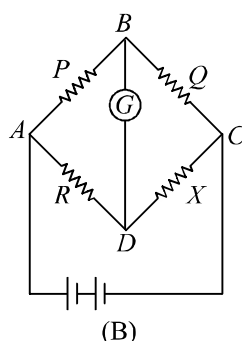
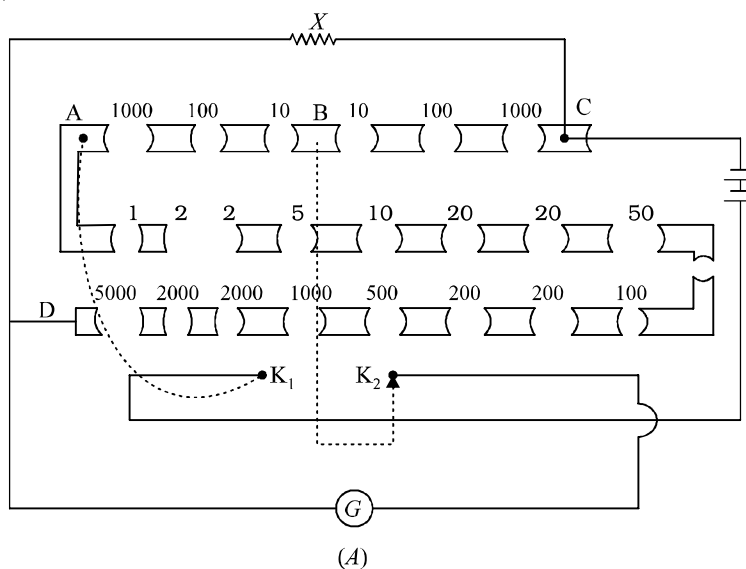
It is clear that $R_1 > R_2$ when $l > 50\text{ cm}$, i.e. the balance point is in the second half

And, $R_1 < R_2$ when $l < 50\text{ cm}$, i.e. the balance point is in the first half

MEASUREMENT OF UNKNOWN RESISTANCE USING A POST OFFICE BOX

Experiment - 10

Post office box is useful to measure the value of unknown resistance correctly upto 2nd decimal place. It is based on Wheatstone bridge.



Systematic diagrams of post office box are shown in the above figures. Each of the arms AB and BC contain three resistances of 10Ω , 100Ω and 1000Ω . These arms are generally known as ratio arms. With the help these resistances, we can introduce resistance P in arm AB and resistance Q in arm BC . The resistance arm AD is a complete resistance box containing resistances from 1Ω to 5000Ω . In this arm, we can introduce resistance R by taking out plugs of suitable values. The unknown resistance X is connected in forth arm CD . These four arms actually form Wheatstone bridge shown in Figure (B). For balanced conditions, no deflection is found in galvanometer.

At balanced condition, $PX = QR$, $X = \frac{QR}{P}$.

Initially, we take $Q = 10\Omega$ and $P = 10\Omega$ to set $\frac{Q}{P} = 1$.

Now put Resistance in part A to D such that the Bridge gets balanced. For this, keep on increasing the resistance with 1Ω interval, check the deflection in Galvanometer by first pressing key K_1 , then Galvanometer key K_2 .

Suppose at $R = 4\Omega$, we get deflection toward left and at $R = 5\Omega$, we get deflection toward right. So, we can say that for bridge balance, R should be between 4 to 5.

Now $X = \frac{Q}{P} R = \frac{10}{10} R = R = 4$ to 5 . So we can estimate that X , should be between 4Ω and 5Ω .

To get closer X , in the second observation, let's choose $\frac{Q}{P} = \frac{1}{10}$ i.e. ($Q = 10\Omega$, $P = 100\Omega$)

Suppose now at $R = 42$, we are getting deflection toward left, and at $R = 43$, deflection is toward right. So $R \in (42, 43)$.

Now $X = \frac{Q}{P} R = \frac{10}{100} R = \frac{1}{10} R$ where $R \in (42, 43)$

So we can estimate that $X \in (4.2, 4.3)$. Now to get further closer, choose $\frac{Q}{P} = \frac{1}{100}$ i.e. ($Q = 10\Omega$, $P = 1000\Omega$)

The observation table is shown below.

No. of Obs.	Resistance in the Ratio arm		Resistance in arm AD(R) (Ohm)	Direction of deflection left or right	Unknown resistance $X = \frac{Q}{P} \times R$ (Ohm)
	AB(P) (Ohm)	BC(Q) (Ohm)			
1.	10	10	4	Left	(4 – 5)
			5	Right	
2.	100	10	40	Left (large)	(4.2 – 4.3)
			50	Right (large)	
			42	Left	
			43	Right	
3.	100	10	420	Left	4.25
			424	Left	
			425	No deflection	
			426	Right	

Errors:

The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating etc.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating).

Illustration - 23 (a) What is the minimum and maximum possible resistance, which can be determined by using post office box ?

(A) 0.01Ω , $1111 \times 10^3\Omega$

(B) 0.1Ω , $5000 \times 10^3\Omega$

(C) 0.01Ω , $500 \times 10^3\Omega$

(D) 0.01Ω , 1111Ω

(b) In an experiment with a post office box, the ratio arms are $1000 : 10$. The value of third resistance is $999W$, find the value of unknown resistance.

(A) $99.9 W$

(B) $999 W$

(C) $9.99 W$

(D) $1000 W$

SOLUTION :**(a)-(A)**

$$\therefore X = \frac{QR}{P} \quad \therefore X_{\min} = \frac{Q_{\min} R_{\min}}{P_{\max}} = \frac{10 \times 1}{1000} = 0.01 \Omega$$

$$X_{\max} = \frac{Q_{\max} R_{\max}}{P_{\min}} = \left(\frac{1000}{10} \right) (R_{\max})$$

$$R_{\max} = 1 + 2 + 2 + 5 + 10 + 20 + 20 + 50 + 100 + 200 \\ + 200 + 500 + 1000 + 2000 + 2000 + 5000 = 11110 \Omega$$

$$\therefore X_{\max} = \left(\frac{1000}{10} \right) (11110) = 1111 \times 10^3 \Omega$$

(b)-(C)

$$\therefore X = \frac{Q}{P} \times R = \frac{10}{1000} \times 999 = 9.99 \Omega$$

Illustration - 24

In a certain experiment if $\frac{Q}{P} = \frac{1}{10}$ and in R , if 192Ω is used we get a deflection toward right, at 193Ω , again toward right but at 194Ω , deflection is toward left. The unknown resistance should lie between :

- (A)** 19.2 to 19.3 Ω **(B)** 19.3 to 19.4 Ω **(C)** 19 to 20 Ω **(D)** 19.3 to 19.4 Ω

SOLUTION : (A)

$$X = \frac{Q}{P} (R) = \frac{1}{10} (192 \leftrightarrow 193)$$

$$\therefore x \in (19.2, 19.3)$$

SUBJECTIVE SOLVED EXAMPLES

Example - 1

- (a) If $y = \ln x$ and percentage error in x is 5%, find error in y .
- (b) If $z = \ln x^2 y$ and percentage errors in x and y are 1% and 2%, respectively, find error in z .
- (c) If $y = \tan \theta$ the value of $\theta = 60^\circ$ and error in measurement of angle is 1.8° , find the value of Δy .

SOLUTION :

(a) $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ or $\Delta y = \frac{\Delta x}{x} = 0.05$; So, the absolute error in $y = 0.05$

(b) We are given that $z = \ln(x^2 y)$

Therefore, $dz = \left(\frac{1}{x^2 y} \right) (y(2x dx) + x^2 (dy))$

So, $\Delta z = 2 \left(\frac{\Delta x}{x} \right) + \left(\frac{\Delta y}{y} \right) = 2 \times 0.01 + 0.02 = 0.04$

Therefore, the absolute error in z is 0.04

(c) $y = \tan \theta$ or $\frac{dy}{d\theta} = \sec^2 \theta$ or $dy = \sec^2 \theta d\theta$ or $\Delta y = \sec^2 \theta \Delta \theta$

$$= \sec^2 60^\circ \times 1.8 \times \frac{\pi}{180} = 4 \times 1.8 \times \frac{\pi}{180} = 0.1256 = 0.126$$

Example - 2

The main scale of a Vernier callipers instrument is graded in divisions of 1 mm each, and 20 divisions of the Vernier scale correspond to 19 divisions of the main scale. There is no zero error in the instrument. If the 45th division of the main scale coincides with the 13th division of the Vernier scale, what is the reading ?

SOLUTION :

We know that the least count,

Because 20 divisions of the Vernier scale correspond to 19 divisions of the main scale, if we move backward (from the point where the two scales coincide) by 13 divisions on the Vernier scale, the number of divisions crossed on the main scale will be between 12 and 13. Hence, the zero of the Vernier scale is ahead of the $(45 - 13) = 32^{\text{nd}}$ division of the main scale.

So, the reading is: $R = 32(1) + 13(0.05) = 32.65 \text{ mm}$

Example - 3

During determination of speed of sound in air by resonance column method, the percentage error in measurement of resonance air column length is 2% and that in measurement of frequency is 5%. Find the percentage error in measurement of speed of sound in air. Assume that first resonance air column length is 40 cm and that of second resonance length is 60 cm.

SOLUTION :

$\therefore V = 2f(l_2 - l_1)$

$\therefore \frac{\Delta V}{V} = \frac{\Delta f}{f} + \frac{\Delta(l_2 - l_1)}{l_2 - l_1}$ or $\frac{\Delta V}{V} = \frac{\Delta f}{f} + \frac{\Delta l_2}{l_2 - l_1} + \frac{\Delta l_1}{l_2 - l_1} = \frac{\Delta f}{f} + \frac{\Delta l_2}{20} + \frac{\Delta l_1}{20}$

Here, $\frac{\Delta l_2}{l_2} \times 100 = 2 \quad \therefore \quad \frac{\Delta l_2}{60} \times 100 = 2 \quad \therefore \quad \Delta l_2 = 1.2 \text{ cm}$

and $\frac{\Delta l_1}{l_1} \times 100 = 2 \quad \text{or} \quad \Delta l_1 = \frac{2 \times 40}{100} = 0.80 \text{ cm}$

Percentage error in speed of sound is

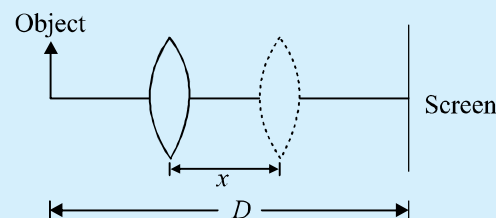
$$\frac{\Delta V}{V} \times 100 = \frac{\Delta f}{f} \times 100 + \frac{\Delta l_2}{20} \times 100 + \frac{\Delta l_1}{20} \times 100 = 5 + \frac{1.2}{20} \times 100 + \frac{0.80}{20} \times 100 = 5 + 6 + 4 = 15\%$$

Example - 4

Displacement method is used to determine the focal length

of convex lens. In this method, the applicable formula is $f = \frac{D^2 - x^2}{4D}$.

Here, D = distance between object plane and screen on which image is formed, f = focal length of lens, x = distance between two position of lens. To determine focal length of lens, the measured value of D and x are 90.0 cm and 30.0 cm, respectively. Find the percentage error in measurement of focal length.

**SOLUTION :**

We are given that $f = \frac{D^2 - x^2}{4D}$

Therefore, $\log(f) = \log\left(\frac{D^2 - x^2}{4D}\right)$

Differentiating, $\frac{1}{f} df = \left(\frac{1}{D^2 - x^2}\right) d(D^2 - x^2) - \left(\frac{1}{4D}\right) (4dD)$

$$\Rightarrow \quad \frac{df}{f} = \left(\frac{1}{D^2 - x^2}\right) [2DdD - 2xdx] - \frac{dD}{D}$$

So, $\frac{\Delta f}{f} = \left(\frac{2}{D^2 - x^2}\right) [D\Delta D + x\Delta x] + \frac{\Delta D}{D} \quad \Rightarrow \quad \frac{\Delta f}{f} = \left(\frac{2}{90^2 - 30^2}\right) [(90)(0.1) + (30)(0.1)] + \frac{0.1}{90} = \frac{2}{45}$

Therefore, the percentage error in the focal length is 4.4%.

Example - 5

In laboratory, specific heat capacity of a solid heavier than water and insoluble in it is determined by Regnault's apparatus. In this method, the hot ball whose specific heat is to be determined is gently dropped into a calorimeter containing water. The calorimeter is stirred. Final temperature of mixture is noted.

Let the mass of ball = m_1 , mass of calorimeter + stirrer = m_2 , mass of water = m_3

Specific heat of ball = s_1 , specific heat of calorimeter + stirrer = s_2 , Specific heat of water = s_3

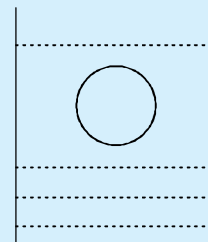
Initial temperature of ball = θ_1 , initial temperature of calorimeter, stirrer and water = θ_2

Final temperature of mixture = θ

Heat lost by the ball $m_1 s_1 (\theta_1 - \theta)$

Heat gained by calorimeter + stirrer = $m_2 s_2 (\theta - \theta_2)$

and heat gained by water = $m_3 s_3 (\theta - \theta_2)$



According to principle of calorimetry, Heat loss = Heat gain

$$\therefore m_1 s_1 (\theta_1 - \theta) = m_2 s_2 (\theta - \theta_2) + m_3 s_3 (\theta - \theta_2) \quad \therefore s_1 = \frac{(m_2 s_2 + m_3 s_3)(\theta - \theta_2)}{m_1 (\theta_1 - \theta)}$$

- (a) Find maximum fractional error in measurement of specific heat of the ball, if m_1, m_2, m_3, s_2, s_3 are precisely known, but least count of temperature measuring thermometer is $\Delta\alpha$.
- (b) If m_1, m_2, m_3, s_2, s_3 are precisely known, then find the value of θ (in terms of θ_1 and θ_2) for which the maximum fractional error in the measurement of s_1 is least.

SOLUTION :

$$(a) \quad \therefore s_1 = \frac{(m_2 s_2 + m_3 s_3)(\theta - \theta_2)}{m_1 (\theta_1 - \theta)}$$

$$\begin{aligned} \therefore \frac{\Delta s_1}{s_1} &= \frac{\Delta(m_2 s_2 + m_3 s_3)}{m_2 s_2 + m_3 s_3} + \frac{\Delta(\theta - \theta_2)}{\theta - \theta_2} + \frac{\Delta\{m_1 (\theta_1 - \theta)\}}{m_1 (\theta_1 - \theta)} = \frac{\Delta(m_2 s_2) + \Delta(m_3 s_3)}{m_2 s_2 + m_3 s_3} + \frac{\Delta\theta + \Delta\theta_2}{\theta - \theta_2} + \frac{\Delta\{m_1 (\theta_1 - \theta)\}}{m_1 (\theta_1 - \theta)} \\ &= \frac{s_2 \Delta m_2 + m_2 \Delta s_2 + s_3 \Delta m_3 + m_3 \Delta s_3}{m_2 s_2 + m_3 s_3} + \frac{\Delta\theta + \Delta\theta_2}{\theta - \theta_2} + \frac{(\theta_1 - \theta) \Delta m_1 + m_1 \Delta(\theta_1 - \theta)}{m_1 (\theta_1 - \theta)} \\ &= \frac{s_2 \Delta m_2 + m_2 \Delta s_2 + s_3 \Delta m_3 + m_3 \Delta s_3}{m_2 s_2 + m_3 s_3} + \frac{\Delta\theta + \Delta\theta_2}{\theta - \theta_2} + \frac{(\theta_1 - \theta) \Delta m_1 + m_1 (\Delta\theta_1 + \Delta\theta)}{m_1 (\theta_1 - \theta)} \end{aligned}$$

If m_1, m_2, m_3, s_2, s_3 have no errors. Then, $\Delta m_1 = \Delta m_2 = \Delta m_3 = 0$ and $\Delta s_2 = \Delta s_3 = 0$

$$\therefore \frac{\Delta s_1}{s_1} = \frac{\Delta\theta + \Delta\theta_2}{(\theta - \theta_2)} + \frac{\Delta\theta_1 + \Delta\theta}{(\theta_1 - \theta)}$$

If temperature measuring thermometer has least count $\Delta\alpha$

$$\begin{aligned} \text{Then, } \Delta\theta = \Delta\theta_2 = \Delta\theta_1 = \Delta\alpha \quad \therefore \frac{\Delta s_1}{s_1} &= \frac{2\Delta\alpha}{\theta - \theta_2} + \frac{2\Delta\alpha}{\theta_1 - \theta} \\ &= 2\Delta\alpha \left(\frac{\theta_1 - \theta + \theta - \theta_2}{(\theta_1 - \theta)(\theta - \theta_2)} \right) = 2\Delta\alpha \frac{(\theta_1 - \theta_2)}{(\theta_1 - \theta)(\theta - \theta_2)} \end{aligned}$$

$$(b) \quad \therefore \frac{\Delta s_1}{s_1} = 2\Delta\alpha \frac{(\theta_1 - \theta_2)}{(\theta_1 - \theta)(\theta - \theta_2)} \quad \therefore \frac{\Delta s_1}{s_1} \text{ is least if } (\theta_1 - \theta)(\theta - \theta_2) \text{ is maximum}$$

$$\text{Let } x = (\theta_1 - \theta)(\theta - \theta_2) \quad \text{or} \quad \frac{dx}{d\theta} = (\theta_1 - \theta) - (\theta - \theta_2)$$

$$\therefore \text{For maximum value, } \frac{dx}{d\theta} = 0$$

$$0 = \theta_1 + \theta_2 - 2\theta \quad \therefore \theta = \frac{\theta_1 + \theta_2}{2}$$

MISCELLANEOUS EXERCISE

Chose the correct alternative. Only one choice is correct :

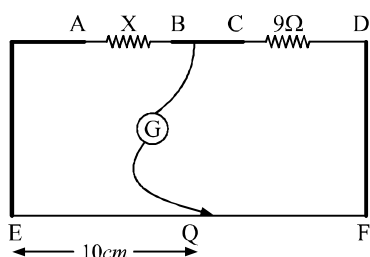
1. A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then possible values of n and m are :

(A) $n = 10, m = 0.5$ cm (B) $n = 9, m = 0.4$ cm
(C) $n = 10, m = 0.8$ cm (D) $n = 10, m = 0.2$ cm

2. In a meter bridge set up, which of the following should be the properties of the one meter long wire ?

(A) High resistivity and low temperature coefficient
(B) Low resistivity and low temperature coefficient
(C) Low resistivity and high temperature coefficient
(D) High resistivity and high temperature coefficient

3. Consider the meter bridge shown in the figure, let the resistance X have temperature coefficient α_1 and the resistance from the resistance box have the temperature coefficient α_2 . Let the reading of the meter scale be 10cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point ? Neglect all the other changes in the bridge due to temperature rise.



(A) $9(\alpha_1 - \alpha_2)\Delta T$ (B) $9(\alpha_1 + \alpha_2)\Delta T$
(C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$ (D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

4. In a Searle's experiment for determination of Young's Modulus, when a load of 50 kg is added to a 3 meter long wire, micrometer screw having pitch 1 mm needs to be given a quarter turn in order to restore the horizontal position of spirit level. Young's modulus of the wire if its cross sectional area is 10^{-5} m^2 is :

(A) $6 \times 10^{11} \text{ N/m}^2$ (B) $1.5 \times 10^{11} \text{ N/m}^2$
(C) $3 \times 10^{11} \text{ N/m}^2$ (D) None

5. On the basis of detail given about two measuring instruments, select the correct statement.

I. Vernier callipers having main scale division = 0.05 cm and Vernier scale division = $\frac{2.45}{50}$ cm

II. Screw guage having pitch 0.5 mm and its circular scale division measures 0.01 mm

(A) Both the instruments have same least count
(B) Least count of screw gauge is lesser than that of vernier calliper
(C) Both the instruments have same least count but screw gauge is more precise
(D) Both the instruments have different least count and screw gauge is more precise

6. In Searle's apparatus, when experimental wire is loaded and unloaded, the air bubble in spirit level gets shifted:

(A) towards reference wire while loading and towards experimental wire while unloading
(B) towards experimental wire while loading and towards reference wire while unloading
(C) towards experimental wire, both the times, during loading and unloading
(D) towards reference wire, both the times during loading and unloading

7. A student is experimenting with resonance tube apparatus in Physics lab to find the speed of sound at room temperature. He got resonating lengths of air column as 17 cm and 51 cm, using tuning fork of frequency 512 Hz. Find speed of sound at room temperature and specify, whether the side water reservoir was moved upward or downward to obtain the second resonance (51 cm)?

(A) 348 m/s, downwards
(B) 348 m/s, upwards
(C) 332 m/s, downwards
(D) 332 m/s, upwards

8. When the gap is closed without placing any object in the screw gauge whose least count is 0.005 mm, the 5th division on its circular scale coincides with the reference line on main scale and zero of main scale is not hidden. When a small sphere is placed, reading on main scale advances by 4 divisions, and the 25th division of the circular scale coincides with the reference line on the main scale. There are 200 divisions on the circular scale. The radius of the sphere is :
- (A) 4.10 mm (B) 4.05 mm
(C) 2.10 mm (D) 2.05 mm
9. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively. Maximum permissible error in area measurement is :
- (A) $\pm 0.2 \text{ cm}^2$ (B) $\pm 0.1 \text{ cm}^2$
(C) $\pm 0.3 \text{ cm}^2$ (D) Zero
10. The least count of a stopwatch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is:
- (A) 16% (B) 0.8%
(C) 1.8% (D) 8%
11. The main scale of a vernier callipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other, the seventh division of the vernier scale coincide with a scale divisions and the zero of the vernier lies to the right of the zero of main scale. Furthermore when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm and the fourth vernier division coincides with a scale division. What is the length of the cylinder?
- (A) 3.14 cm (B) 3.07 cm
(C) 3.2 cm (D) 3.04 cm
12. The length of a cylinder is measured with vernier callipers whose smallest division on main scale is 0.5 mm and 10 vernier divisions is equal to 9 main scale divisions. It is observed that 80th division of the main scale coincides with the 4th division of the vernier scale. The length is:
- (A) 4.05 cm (B) 3.63 cm
(C) 4.10 cm (D) 3.82 cm
13. The distance advanced by screw of a screw gauge is 2 mm in four rotation. Its cap is divided into 50 division. There is no zero error. If the screw reads 3 divisions on the main scale and 32 divisions on the cap, then the diameter of the wire is:
- (A) 3.32 mm (B) 1.82 mm
(C) 2.82 mm (D) 4.7 mm
14. A YDSE experiment is used to measure the wavelength of light used. The distance between screen and slits is measured by a meter scale with least count 1 mm. The measured distance is 1.500 m. The distance between the slits is measured by a micrometer of least count 0.01 mm. The distance between slits is measured to be 0.33 mm. The fringe width is measured by the same micrometer. It comes out to be 2.50 mm. the wavelength of light is:
- (A) $550 \pm 19 \text{ nm}$ (B) $550 \pm 39 \text{ nm}$
(C) $330 \pm 28 \text{ nm}$ (D) $330 \pm 56 \text{ nm}$
15. In Searle's experiment to find Young's modulus, the diameter of wire is measured as $D = 0.05 \text{ cm}$, length of wire is $L = 125 \text{ cm}$. When a weight, $m = 20.0 \text{ kg}$ is put, extension in wire was found to be 0.100 cm. Find maximum permissible error in Young's modulus.
- (A) 3.3% (B) 4.3%
(C) 5.5% (D) 2.5%
16. **Assertion :** In Searle's experiment to find Young's modulus, a reference wire is also used along with the experiment wire.
Reason : Reference wire neutralizes the effect of temperature, yielding of support and other external factors.
- (A) If both assertion and reason are true and the reason is a correct explanation of assertion
(B) If both assertion and reason are true but reason is not a correct explanation of assertion
(C) If assertion is true but reason is false
(D) If both assertion and reason are false

17. In optical bench experiment, index error is +1 cm and -1 cm, between object needle and lens, lens and image needle respectively. Observed values of u and v are 9 cm and 17 cm. Focal length of lens is:
 (A) 6.15 cm (B) 5.54 cm
 (C) 5.88 cm (D) 6.25 cm
18. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance then :
 (A) $x > 54$ (B) $54 > x > 36$
 (C) $36 > x > 18$ (D) $18 > x$
19. In an experiment to determine the value of g using a simple pendulum, the error in the length of the pendulum string is 4%, and the error in the time period is 0.5%. The error in the value of g is:
 (A) 4.5% (B) 5%
 (C) 3.5% (D) 3%
20. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C . The final temperature is 40°C . The specific heat of the metal is:
 (A) $0.1\text{ Jg}^{-1}\text{ }^\circ\text{C}^{-1}$ (B) $0.2\text{ Jg}^{-1}\text{ }^\circ\text{C}^{-1}$
 (C) $0.3\text{ Jg}^{-1}\text{ }^\circ\text{C}^{-1}$ (D) $0.4\text{ Jg}^{-1}\text{ }^\circ\text{C}^{-1}$

ANSWERS TO MISCELLANEOUS EXERCISE

1. C	2. A	3. A	4. A	5. A	6. A	7. A	8. A	9. A	10. B
11. B	12. D	13. B	14. A	15. B	16. A	17. B	18. A	19. B	20. A